considered will be assumed throughout this paper. The case of noisy, partially masked maps will be addressed in future studies.

In Section 2, we review the estimator of the (binned) angular bispectrum and $f_{\rm NL}$ and develop a parametrization of the bispectrum to display and visualize it efficiently. In Section 3, we develop a prescription to infer the bispectrum from the power spectrum for clustered sources and for different populations. In Section 4, we use publicly available full-sky simulations of radio and infrared sources to compute and characterize their bispectrum at CMB frequencies and we compare them to the predictions from the prescription. We examine the configuration dependence of the point-source bispectra and study the bias they induce on the estimation of the primordial local NG in Section 5. We finally conclude and discuss our results in Section 6.

2 THREE-POINT NG ESTIMATORS

2.1 Full-sky angular bispectrum estimator

Given a full-sky map of the temperature fluctuations $\Delta T(\boldsymbol{n})$ of some signal, it can be decomposed in the spherical harmonic basis

$$a_{\ell m} = \int \mathrm{d}^2\boldsymbol{n}\; Y^*_{\ell m}(\boldsymbol{n})\; \Delta T(\boldsymbol{n}) \tag{1}$$

with the usual orthonormal spherical harmonics $Y_{\ell m}$,

$$\int \mathrm{d}^2\boldsymbol{n}\; Y_{\ell m}(\boldsymbol{n})\; Y^*_{\ell' m'}(\boldsymbol{n}) \;=\; \delta_{\ell\ell'}\,\delta_{mm'} .$$

Observational data are pixelized, so that the integral is replaced by a sum over pixels. We will assume that the solid angle of a pixel, $\Omega_{\rm pix}$, is a constant, which is for example the case for the HEALPIX[1] pixelization scheme that we will adopt for the numerical calculations. In this case we have

$$a_{\ell m} = \sum_{\boldsymbol{n}_i} \; Y^*_{\ell m}(\boldsymbol{n}_i)\; \Delta T(\boldsymbol{n}_i)\, \Omega_{\rm pix} \, . \tag{2}$$

This discreteness effect will be important e.g. in Section 3.1.

In order to compute the angular bispectrum, which is the harmonic transform of the three-point correlation function, we will resort to scale maps as defined by Spergel & Goldberg (1999) and also used by Aghanim et al. (2003) and De Troia et al. (2003):

$$T_\ell(\boldsymbol{n}) = \sum_m a_{\ell m} Y_{\ell m}(\boldsymbol{n}) = \int \mathrm{d}^2\boldsymbol{n}'\; \Delta T(\boldsymbol{n}')\; P_\ell(\boldsymbol{n}\cdot\boldsymbol{n}'), \tag{3}$$

where P_ℓ is the Legendre polynomial of order ℓ.

The optimal bispectrum estimator is then (Spergel & Goldberg 1999)

$$\hat{b}_{\ell_1\ell_2\ell_3} = \frac{4\pi}{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}^{-2} \times \int \mathrm{d}^2\boldsymbol{n}\; T_{\ell_1}(\boldsymbol{n})\, T_{\ell_2}(\boldsymbol{n})\, T_{\ell_3}(\boldsymbol{n}) \tag{4}$$

or it can be written in the form

$$\hat{b}_{\ell_1\ell_2\ell_3} = \sqrt{\frac{4\pi}{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}^{-1} \times \sum_{m_1,m_2,m_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} a_{\ell_1 m_1}\, a_{\ell_2 m_2}\, a_{\ell_3 m_3}, \tag{5}$$

where the expression in brackets represents the Wigner $3j$ symbols. Equation (5) is computationally expensive when implemented at high ℓ due to the large number of Wigner symbols to calculate. Equation (4) still requires a few cpu-days for a full computation at a *Planck*-like resolution, Nside = 1024–2048. Binning the multipoles in ℓ, as in Bucher, Tent & Carvalho (2010), has the advantage of speeding up the computations and smoothing out the variations due to cosmic variance.

For a given triangle in harmonic space (ℓ_1, ℓ_2, ℓ_3) the number of independent configurations on the sphere is

$$N_{\ell_1\ell_2\ell_3} = \frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ 0 & 0 & 0 \end{pmatrix}^2 . \tag{6}$$

When multipoles are binned in bins of width $\Delta\ell$ the expression for the scale maps (equation 3) becomes

$$T_{\Delta\ell}(\boldsymbol{n}) = \sum_{\ell\in\Delta\ell, m} a_{\ell m}\; Y_{\ell m}(\boldsymbol{n}) \tag{7}$$

and a binned bispectrum estimator identically weighting triangles is given by

$$\hat{b}_{\Delta\ell_1,\Delta\ell_2,\Delta\ell_3} = \frac{1}{N_\Delta(\Delta\ell_1, \Delta\ell_2, \Delta\ell_3)} \int \mathrm{d}^2\boldsymbol{n}\; T_{\Delta\ell_1}(\boldsymbol{n})\, T_{\Delta\ell_2}(\boldsymbol{n})\, T_{\Delta\ell_3}(\boldsymbol{n}), \tag{8}$$

where

$$N_\Delta(\Delta\ell_1, \Delta\ell_2, \Delta\ell_3) = \sum_{\ell_i\in\Delta\ell_i} N_{\ell_1\ell_2\ell_3} .$$

One can easily check that the obtained binned bispectrum estimator is unbiased for a constant bispectrum and that the bias can be neglected as long as the bispectrum does not vary significantly within a bin $\Delta\ell$. In the following, we have chosen $\ell_{\max} = 2048$ and a bin width $\Delta\ell = 64$ for simplicity and computational speed while retaining enough information on the scale dependence (Bucher et al. 2010).

2.2 $f_{\rm NL}$ estimator

The most studied and constrained form of primordial NG is the local ansatz, whose amplitude is parametrized by a non-linear coupling constant $f_{\rm NL}$:

$$\Phi(\boldsymbol{x}) = \Phi_{\rm G}(\boldsymbol{x}) + f_{\rm NL}\left(\Phi^2_G(\boldsymbol{x}) - \langle\Phi^2_{\rm G}(\boldsymbol{x})\rangle\right), \tag{9}$$

where $\Phi(\boldsymbol{x})$ is the Bardeen potential and $\Phi_{\rm G}(\boldsymbol{x})$ is a Gaussian field. This form of NG yields the following CMB angular bispectrum (Komatsu & Spergel 2001):

$$b^{\rm loc}_{\ell_1\ell_2\ell_3} = \int r^2\,\mathrm{d}r\; \alpha_{\ell_1}(r)\, \beta_{\ell_2}(r)\, \beta_{\ell_3}(r) + \text{permutation}, \tag{10}$$

with

$$\alpha_\ell(r) = \frac{2}{\pi}\int k^2\mathrm{d}k\; g_{{\rm T},\ell}(k)\; j_\ell(kr) \tag{11}$$

$$\beta_\ell(r) = \frac{2}{\pi}\int k^2\mathrm{d}k\; P(k)\, g_{T,\ell}(k)\; j_\ell(kr), \tag{12}$$

where $g_{{\rm T},\ell}$ is the radiation transfer function, which can be computed with a Boltzmann code, j_ℓ are the spherical Bessel functions and $P(k) \propto k^{n_{\rm s}-4}$ is the primordial power spectrum, with a spectral index $n_{\rm s}$.

On large angular scales, the SW effect is the dominant contribution to the CMB signal. In this regime, the CMB bispectrum takes the following analytical form:

$$b^{\rm loc}_{\ell_1\ell_2\ell_3} \propto -\left(\frac{1}{\ell_1^2\,\ell_2^2} + \frac{1}{\ell_1^2\,\ell_3^2} + \frac{1}{\ell_2^2\,\ell_3^2}\right) . \tag{13}$$

[1] http://healpix.jpl.nasa.gov